gyro parameters and inputs for t_i equal to 300 sec is

$$|\alpha_e| \approx \frac{1}{2}\frac{H_W\Omega_{SA}\Omega_{IA}}{K}\,t_i = \frac{1}{2}\,\frac{(9\times 10^4)(1)(1)(300)}{(57.3)(57.3\times 10^4)}$$

$$|\alpha_e| \approx 0.41°$$

The error will increase (or decrease) in proportion to the product of the amplitudes ($\Omega_{SA}\Omega_{IA}$) of the rates.

Reference

[1] "A handbook on floated integrating gyros," Reeves Corp., p. 8 (1958).

A Thermal Boundary-Layer Problem in Magnetohydrodynamics

John Fillo*
Syracuse University, Syracuse, N. Y.

Nomenclature

C_p = coefficient of specific heat at constant pressure
$\mathbf{H}$ = magnetic-field vector, magnitude H
h_x, h_y = components of H along x, y
Hx, Hy = dimensionless components, $Hx = h_x/H_\infty$, $Hy = h_y/H_\infty$
H_∞ = magnetic-field strength in undisturbed stream
$\mathbf{j}$ = electric-current-density vector, magnitude j
k = thermal conductivity
L = reference length
Pm = $H_\infty{}^2/4\pi\ C_p\ T_\infty$
Pr = Prandtl number, uC_p/k
Prm = magnetohydrodynamic Prandtl number, $4\pi\sigma\nu$
$\mathbf{q}$ = velocity vector
Re = Reynolds number, UL/ν
Rm = magnetic Reynolds number, $4\pi\ U\sigma L$
T = temperature
T_∞ = temperature in undisturbed stream
U_∞ = undisturbed-stream speed
u, v = components of q in x, y directions
U, V = dimensionless components, $U = u/U_\infty$, $V = v/U_\infty$
x, y = boundary-layer coordinates
X, Y = dimensionless coordinates, $X = x/L$, $Y = y/\delta_t L$
δ_t = typical thermal boundary-layer thickness, dimensionless
ν = kinematic viscosity
ρ = mass density
σ = electrical conductivity
θ = dimensionless temperature, $\theta = T - T_\infty/T_\infty$

Introduction

In Ref. 1, Sears studied a class of steady plane and axisymmetric magnetohydrodynamic flows known as the aligned-fields flow, i.e., flow in which the basic or undisturbed situation consists of a uniform parallel stream and a uniform magnetic field parallel to it.

The flow regime was split into three regions, namely, the potential flow, the inviscid layer where magnetohydrodynamics (MHD) effects are present, and a viscous sublayer. The boundary-layer equations pertinent to the inviscid magnetohydrodynamic boundary layer, as well as the viscous sublayer, were derived and discussed. Similar solutions were found for these equations. The object of this note is to derive and examine the thermal boundary-layer equations for steady plane aligned-fields flow.

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* Instructor, Department of Mechanical and Aerospace Engineering.

Analysis

The thermal boundary-layer equations are derived for fluids subject to large magnetic Reynolds numbers, small magnetic Prandtl numbers, and Prandtl numbers of order one. In addition, we shall only consider constant-property fluids and incompressible flow.

The temperature distribution about a heated body may be found from the following equation:

$$\rho C_p(\mathbf{q}\cdot\nabla T) = k\nabla^2 T + \rho\nu\Phi + j^2/\sigma \tag{1}$$

where T is the local temperature, $\mathbf{q}$ is the local velocity vector with components u, v and is assumed to be known from Ref. 1, Φ is the viscous dissipation, and j^2/σ is the Joule heating term.

In addition to the velocity vector $\mathbf{q}$, we need to know $\mathbf{j}$. The necessary equations are

$$\mathbf{j} = \sigma(\mathbf{q}\times\mathbf{H}) \tag{2}$$

$$4\pi\mathbf{j} = \text{curl}\,\mathbf{H} \tag{3}$$

Where electromagnetic units are employed, the electric field E is taken to be zero, and the fluid is assumed to be nonmagnetic.

In order to talk about the thermal boundary layer, we must take into account the fact that the flow regime is split into three regions. In light of this, the following model for the study of the thermal effects about a solid body is proposed: 1) an undisturbed region where the temperature is T_∞; 2) a magnetoinviscid-thermal layer where thermal conduction is negligible, and only convection and Joule heating contribute to the transport of thermal energy; and 3) a viscous-thermal layer where thermal conduction, convection, viscous, and Joule heating are important. This idea of splitting the thermal region was also considered in Ref. 2 but for an entirely different problem.

The justification for neglecting the heat transport by conduction in the magnetoinviscid-thermal layer rests on the fact that the fluids under consideration are those for which the heat conductivity is small. Thus, it is well known[3] that conduction becomes important only in the region where the convective heat transport is small because of small velocities, and this occurs in the region near the solid surface of the body, since the velocity goes to zero.

Magneto-inviscid-thermal layer

To derive the boundary-layer equations pertinent to this region, let us consider the following transformation to dimensionless variables:

$$X = x/L \qquad Y = y/L\delta_t$$

where $L\delta_t$ is a typical thickness of the boundary layer. In dimensionless form, Eq. (1) becomes

$$U\frac{\partial\theta}{\partial X} + \frac{V}{\delta_t}\frac{\partial\theta}{\partial Y} = \frac{Pm}{Rm}\left(\frac{\partial Hy}{\partial X} - \frac{1}{\delta_t}\frac{\partial Hx}{\partial Y}\right)^2 \tag{4}$$

As in Ref. 1 where all dependent variables are expressed in the form of a power series, we now assume that the nondimensional temperature θ may be expressed in the form of a power series in a small parameter that vanishes in the limit $Rm = \infty$, i.e.,

$$\theta = \sum_{n=0}^{\infty}\theta_n X^n \tag{5}$$

where

$$X^n = Rm^{-P} \qquad P > 0 \tag{6}$$

Absorbing the constant of proportionality in the definition of δ_t, we assume that the dimensionless thermal boundary-layer thickness may be written as

$$\delta_t = Rm^{-Q} \qquad Q > 0 \tag{7}$$

and vanishes in the limit $Rm = \infty$.

Although the formulations of Eqs. (5–7) appear to be arbitrary, as was the case of the inviscid boundary-layer thickness, velocity, and magnetic-field components of Ref. 1, nevertheless, they lead to consistent results.

The results from Ref. 1 which we need in our analysis are

$$\begin{aligned} U &= U_0 + U_1 Rm^{-1/2} + \ldots \\ V &= V_1 Rm^{-1/2} + \ldots \\ Hx &= Hx_0 + Hx_1 Rm^{-1/2} + \ldots \\ Hy &= Hy_1 Rm^{-1/2} + \ldots \end{aligned} \tag{8}$$

Introducing the set of equations [Eqs. (8)] as well as (5–7) into Eq. (4), we have

$$U_0 \frac{\partial\theta_0}{\partial Y} + \ldots + V_1 \frac{\partial\theta_0}{\partial Y} Rm^{(Q-1/2)} + \ldots = \frac{Pm}{Rm}\left[\frac{\partial Hy_1}{\partial X} Rm^{-1/2} + \ldots - \frac{\partial Hx_0}{\partial X} Rm^{+Q} \ldots\right]^2 \tag{9}$$

Thus it appears that, for $Q = \frac{1}{2}$, we are led to an equation, the coefficient of Rm^0 of boundary-layer character

$$U_0 \frac{\partial\theta_0}{\partial X} + V_1 \frac{\partial\theta_0}{\partial Y} = Pm \left[\frac{\partial Hx_0}{\partial Y}\right]^2 \tag{10}$$

Returning to the original dimensional variables, the magnetoinviscid-thermal boundary-layer equation is

$$\rho C_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y}\right) = \frac{1}{(4\pi)^2\sigma}\left(\frac{\partial h_x}{\partial y}\right)^2 \tag{11}$$

with the boundary condition $T = T_\infty$ for large y. Note that the temperature at the inner edge of the inviscid-thermal layer must be determined from the solution of Eq. (11).

Viscous-thermal sublayer

For the viscous-thermal region, we must examine all of the terms of Eq. (1). Before doing this, let us look at some of the important results from Ref. 1 for the viscous sublayer. First of all, it was shown that the viscous boundary layer with magnetic effects was of order $Re^{-1/2}$. Also, H_x is negligible, and H_y may be taken as constant across the viscous sublayer, its value having been calculated for the inner edge of the inviscid boundary layer. In addition,

$$j = -\tfrac{1}{4}(\partial h_x/\partial y) = \sigma u h_y \tag{12}$$

We see, then, that the boundary-layer analysis for the viscous-thermal region does not differ from the nonmagnetic case. The thermal boundary layer is of order $Re^{-1/2}Pr^{-1/2}$, and Eq. (1) becomes

$$\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\frac{\partial^2 T}{\partial y^2} + \rho\nu\left(\frac{\partial u}{\partial y}\right)^2 + \sigma u^2 h_y^2 \tag{13}$$

where u and v are the solutions of the viscous sublayer equations. The boundary condition is $T = T_w$ at the surface of the body and is prescribed; at the outer edge of the viscous-thermal layer, $T = T_i$, where T_i is found from the inner edge of the inviscid-thermal layer.

Equation (13) is nothing more than the thermal-energy equation that has been studied by Rossow,[4] Lykoudis,[5] and others in their study of boundary layers at low Rm with the applied magnetic field directed normal to the body surface. It is to be treated, though, after the inviscid-thermal boundary-layer equation [Eq. (11)] has been solved.

In conclusion, Eq. (11) is being studied now in terms of the similarity variable that was introduced by Sears in Ref. 1. What is needed, before a complete analysis can be made, is further study of the velocity and magnetic-field components as has been initiated by Sears and Mori.[6]

References

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Plane Poiseuille Flow of a Radiating and Conducting Gas

Ralph Greif*

University of California, Berkeley, Calif.

Nomenclature

k = absorption coefficient
T^* = temperature
T_0^* = reference temperature, taken to be T_w^*
T = dimensionless temperature, T^*/T_0^*
y = distance from lower wall
q^* = total heat flux
q = dimensionless heat flux, $q^*/\sigma T_0^{*4}$
λ = thermal conductivity
τ^* = optical depth, $\int_0^y k\,dy$
τ = $3\tau^*/2$
σ = Stefan-Boltzmann constant
μ = viscosity
u = velocity
ϵ = $3\lambda k/4T_0^{*3}\tau_w^2$
δ = $\epsilon\tau_w^2$
ξ = y/y_w
ψ = $\mu u_M^2/y_w\sigma T_0^{*3}$

Subscripts

R = radiation
1 = lower wall
w = upper wall

In a previous paper,[1] the problem of Couette flow with a radiating and conducting gas was solved. It should also be pointed out that the rather broad class of radiation and conduction heat-transfer problems, which obey an energy equation of the form

$$(d/dy)[-\lambda(dT^*/dy) + q_R^*] = f(y) \tag{1}$$

that is, with variable dissipation, heat sources, etc., may also

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* Assistant Professor of Mechanical Engineering. Member AIAA.